Precedence rules for operators

- $\sim$ (negation) with highest priority amongst logical operators.
- $\wedge, \vee, \rightarrow, \leftrightarrow$ in decreasing order of priority.
- Propositional expressions are evaluated from left to right.
- ~~Properties of~~ priorities of adjacent operators (OP1 and OP2) are compared. If priority (OP1) $\geq$ priority (OP2) then 'OP' is evaluated else moves to next adjacent operator.
- Expression within parenthesis is evaluated first.

Ex Example $P \wedge Q \rightarrow R \equiv ((P \wedge Q) \rightarrow R)$

To change the order of priority we need to assign parenthesis

$P \wedge (Q \rightarrow R)$

- Evaluation of adjacent operators of same priority is done from left to right.

$P \vee Q \vee R \equiv (P \vee Q) \vee R$

Examples Consider following propositional expression which are fully parenthesized according to the priorities of operators

- $P \wedge Q \vee R \wedge S \equiv ((P \wedge Q) \vee (R \wedge S))$

- $P \vee Q \vee R \wedge S = ((P \vee Q) \vee (R \wedge S))$

- $P \vee Q \wedge R \rightarrow \sim S \rightarrow \cancel{Q}T$

  $\cancel{\equiv ((P \vee Q)}$

  $= (((P \vee (Q \wedge R)) \rightarrow (\sim S)) \rightarrow T)$

- $P \rightarrow Q \vee R \Leftrightarrow \sim S \wedge T$

  $= ((P \rightarrow (Q \vee R)) \Leftrightarrow ((\sim S) \wedge T))$

- $(P \rightarrow Q) \vee (R \vee \sim S) \wedge T$

  $= ((P \rightarrow Q) \vee ((R \vee (\sim S)) \wedge T))$

Example Compute the truth values of a formula. $P \vee Q \rightarrow (\sim Q \rightarrow \sim P)$

$$P \vee Q \rightarrow (\sim Q \rightarrow \sim P)$$
$$= \underbrace{\left((P \vee Q) \rightarrow \left((\sim Q) \rightarrow (\sim P) \right) \right)}_{\alpha}$$

P	Q	$\sim Q$	$\sim P$	$P \vee Q$	$\sim Q \rightarrow \sim P$	α
T	T	F	F	T	T	T
T	F	T	F	T	F	F
F	T	F				

Example Evaluate the truth value of the following formulae

$$(P \rightarrow Q) \wedge R \leftrightarrow P \wedge Q \vee S$$

Given that $\{P, Q, R, S\} \rightarrow \{F, T, F T\}$

$$(P \rightarrow Q) \wedge R \leftrightarrow P \wedge Q \vee S$$
$$= \left((P \rightarrow Q) \wedge R \right) \leftrightarrow \left((P \wedge Q) \vee S \right)$$
$$= (T \wedge F) \leftrightarrow (F \vee T)$$
$$= F \leftrightarrow T$$
$$= F$$

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Equivalence laws :→

Equivalence laws are used to reduce / simplify a given formula to another new formula. These laws can be verified using truth table

1. Commutation
   1. $P \wedge Q \cong Q \wedge P$
   2. $P \vee Q \cong Q \vee P$

2. Association

$P \wedge (Q \wedge R) \cong (P \wedge Q) \wedge R$

$P \vee (Q \vee R) \cong (P \vee Q) \vee R$

3. Double Negation

$\sim(\sim P) \cong P$.

4. Distributive laws
   1. $P \wedge (Q \vee R) = (P \wedge Q) \vee (P \wedge R)$
   2. $P \vee (Q \wedge R) = (P \vee Q) \wedge (P \vee R)$

5. De-Morgan's law
   1. $\sim (P \wedge Q) \cong \sim P \vee \sim Q$
   2. $\sim (P \vee Q) \cong \sim P \wedge \sim Q$

6. Law of Excluded Middle.

$P \vee \sim P \cong T$.

7. law of Contradiction

$P \wedge \sim P \cong F$

Exercise: Show that following formulas are logically equivalent using equivalence laws :→

1. $(P \wedge Q) \vee (P \wedge \sim Q) \cong P$

Soln :→ $(P \wedge Q) \vee (P \wedge \sim Q) \cong P \wedge (Q \vee \sim Q)$ (Distributive law)

$\cong P \wedge T$ (Laws of excluded middle)

$\cong P$.

Hence, $(P \wedge Q) \vee (P \wedge \sim Q) \cong P$.

2. $\sim P \rightarrow \sim (P \rightarrow \sim Q) \cong P$

we have ~~A⊕B~~ $\underline{P \rightarrow Q} \cong \sim P \vee Q$

Then $\sim P \rightarrow \sim (P \rightarrow \sim Q)$

$\cong \sim(\sim P) \vee (\sim (P \rightarrow \sim Q))$

$\cong P \vee (\sim (P \rightarrow \sim Q))$ (Double negation.

$\cong P \vee (\sim (\sim P \vee \sim Q))$

~~≅ PV (~(~P) ∧~~

$\cong P \vee (\sim (\sim (P \wedge Q))$ By D'Morgan's law.

Substitution rule :→

let α be any formula and β be any sub formula occurring in α. If λ is another formula such that $\beta \cong \lambda$, then a new formula δ obtained by replacing at least one occurrence of β by λ in α is logically equivalent to α i.e $\alpha \cong \delta$

Example $\alpha : P \to (R \to Q) \Leftrightarrow \sim P \vee (R \to Q)$

Consider sub formula $\beta : R \to Q$ of α.

Now $\beta \cong \sim R \vee Q : \lambda$

Then $\delta : P \to (\sim R \vee Q) \Leftrightarrow \sim P \vee (\sim R \vee Q)$

$\delta : P \to (\sim R \vee Q) \Leftrightarrow \sim P \vee (R \to Q)$

$\delta : P \to (R \to Q) \Leftrightarrow \sim P \vee (\sim R \vee Q)$

are all logically equivalent to α.